Math 42-Number Theory
Problem Set #1
Due Tuesday, February 8, 2011

1. Use Euclid's algorithm to find the GCD of 29 and 11.

2. Write $\frac{29}{11}$ as a simple continued fraction. Compute all the convergents.

3. Find integers x, y such that $29x + 11y = 1$.

4. Describe *all* integer solutions (x, y) to the equation $29x + 11y = 1$.

5. Use Euclid's algorithm to find the GCD of 72 and 25.

6. Write $\frac{72}{25}$ as a simple continued fraction and compute the convergents.

7. Find *natural numbers* x and y such that $72x + 25y = 1776$.

8. In a game where you can score a or b points at a time $(a, b \in \mathbb{N})$, what is the largest unattainable score? Explain why this score is unattainable. (You don't have to show that it is the *largest* unattainable score, just that this score is unattainable.)

9. Given $a, b \in \mathbb{N}$, for which values of $c \in \mathbb{Z}$ can you solve the diophantine equation $ax + by = c$? (*Extra Credit:* Prove your claim. You may use the fact that the smallest natural number expressible in the form $ax + by$ is the GCD of a and b.)

10. Give an example of natural numbers a, b, c such that $a \mid bc$ but $a \nmid b$ and $a \nmid c$. Prove that for $a, b, c \in \mathbb{N}$, if $a \mid bc$ and $(a, b) = 1$, then $a \mid c$. (Hint: What do you know is true if $(a, b) = 1$?)